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DYNAMIC SIZING OF SAFETY STOCK

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ABSTRACT

Generally the safety stock is maintained constant and it is not changed within the planning horizon to follow any demand pattern, in particular the seasonal one. The authors wonder about the correctness of such an approach and thus examine the adoption of dynamic versus static safety stocks.

Two static rules, based on shortage occurrences and service level respectively, and two dynamic ones (the periods of supply rule and a new method proposed by the authors) are compared and products with stationary, linear trend and seasonal demand patterns are involved in the analysis. Simulation experiments are performed. Results clearly show how static rules get better performances than the dynamic ones even in the seasonal case.

1. INTRODUCTION

It is a common practice to keep safety stocks constant and not to change them within the planning horizon to follow any demand pattern, in particular the seasonal one.

The authors wonder about the correctness of such an approach; therefore they try to analyse the adoption of dynamic versus static safety stocks.

Brown (1967) considered three different rules to determine safety stocks. In particular:

1. safety stock as periods of supply:

$$SS = P \cdot \hat{d} \quad (1)$$

where P is the number of periods of supply to be covered and \hat{d} the forecast level of demand per period;

2. safety stock as the product of a safety factor Z^* and the standard deviation σ_L of the error in forecasting the total demand in a lead time:

$$SS = Z^* \cdot \sigma_L \quad (2)$$

The safety factor corresponds to the specified chance α that no shortage occurs before the next shipment arrives into stock, so that $F(Z^*) = 1 - \alpha$, where F is the probability function of the normalised variable $Z = (E_L - \mu_L)/\sigma_L$, being E_L the forecast error and μ_L the forecast error mean;

3. safety stock as a function of a desired level of service LS , meaning the probability of filling the order. Safety stock can still be determined by relation (2), but in this case Z^* is evaluated as:

$$E(Z^*) = (1 - LS) \cdot \frac{Q}{\sigma_L} \quad (3)$$

where E is the partial expectation, so that $\sigma_L \cdot E(Z^*)$ expresses the expected quantity back ordered when a shortage occurs, and Q is the order quantity.

The first rule can be regarded as a dynamic sizing of the safety stock, because demand level pattern is followed, while the last two methods can be considered as a static way of determining safety stocks.

Brown suggested that at an aggregate level, when a whole set of products has to be managed by a company, the third rule performs better, so that a static approach should properly be considered.

We want to deeply analyse the behaviour of an inventory system under different safety stock rules to improve our understanding of dynamic versus static method performances.

Therefore we consider the three "classic" rules described above and a new dynamic rule proposed in the following section.

2. A NEW DYNAMIC RULE

The first rule proposed by Brown, based on periods of supply to be covered, has no direct links with the forecast error a safety stock has to protect against.

Also, it provides no direct relations with the service level offered to customers, the most common performance measure used to analyse inventory systems.

Since our aim is to get a better insight into the behaviour of dynamic versus static rules, we propose another way to dynamically determine safety stocks related both to the forecast error and service level. Our intent is to avoid that the intrinsic limitations of the first rule could distort the comparison between the dynamic/static nature of safety stock sizing.

We propose the following dynamic rule:

$$SS = Z^* \cdot \sigma_L \cdot \frac{\hat{d}}{d_m} \quad (4)$$

where Z^* is the safety factor for a given service level (see relation 3);

σ_L is the standard deviation of error in forecasting demand level during the lead time;

\hat{d} is the forecast demand during a given period;

d_m is the average demand per period of the last year.

3. SIMULATION EXPERIMENTS

To analyse the different performances obtained with the four safety stock rules described above, we simulated an inventory system managed with the TPOP (Time-Phased Order Point) approach and a lot-for-lot sizing.

We were interested on evaluating the behaviour of the system when the four methods are used to settle the safety stock for:

- a set of products with the same demand pattern, to evaluate if the goodness of different sizing rules is affected by demand characteristics;
- a set of products with different demand patterns, so that the performances of the four methods can be compared at an aggregate level. The decision variables (periods of supply,

safety factor and service level for rule 1,2 and 3-4 respectively) take the same value for all products.

For all products a demand history of 120 periods (10 years if one month periods are considered) was generated, basing on the following relations (Hax and Candea, 1984):

- stationary demand distribution

$$d_i = a + \varepsilon_i \quad (5)$$

- demand with linear trend pattern.

$$d_i = a + b \cdot i + \varepsilon_i \quad (6)$$

- demand with seasonal pattern

$$d_i = a + c \cdot \sin\left(\frac{2\pi \cdot i}{12}\right) + \varepsilon_i \quad (7)$$

- demand with linear trend and seasonal pattern

$$d_i = a + b \cdot i + c \cdot \sin\left(\frac{2\pi \cdot i}{12}\right) + \varepsilon_i \quad (8)$$

where d_i = demand in period i ;

a = mean time-invariant demand, set to 1500 units;

ε_i = random noise component in period i , normally distributed with zero mean and a standard deviation σ_ε ;

b = trend component;

c = seasonal component.

Values taken by the above parameters are shown in Table 1.

Tab.1 Values taken by demand distribution parameters

Demand pattern	Number of items	σ_ε	b	c
stationary	30	100	-	-
	30	300		
	30	500		
linear trend	30-3=90	100	10, 30, 50	-
	30-3=90	300	10, 30, 50	
	30-3=90	500	10, 30, 50	
seasonal	30-3=90	100	-	150, 450, 750
	30-3=90	300		150, 450, 750
	30-3=90	500		150, 450, 750
trend and seasonal	30-3-3=270	100	10, 30, 50	150, 450, 750
	30-3-3=270	300	10, 30, 50	150, 450, 750
	30-3-3=270	500	10, 30, 50	150, 450, 750

Since forecasting methodologies generally require a certain period of time for stabilisation, it is reasonable to believe that the performances related to different safety stock sizing rules could be affected by the amount of past data available. Preliminary runs showed that a stabilised condition is achieved in the simulated system after a 30 month period.

In this situation the following forecasting methods were used (Tersine, 1988):

- moving average including 6 periods for stationary demand products;

- b. exponentially weighted moving average ($\alpha=0,2$) with trend correction ($\beta=0,1$) for demand with linear trend pattern;
- c. exponentially weighted moving average ($\alpha=0,2$) with seasonal correction ($\gamma=0,1$) for demand with seasonal pattern;
- d. exponentially weighted moving average ($\alpha=0,2$) with trend correction ($\beta=0,1$) and seasonal correction ($\gamma=0,1$), for demand with both trend and seasonal pattern.

We decided to study not only the behaviour of the four sizing rules in a stabilised condition, but also during the first three years, i.e. a typical nowadays product life cycle, and the first year of product life, when a very unstable condition has to be faced.

To analyse if the replenishment lead time affects simulation results, three values were considered for all runs equal to 1, 2, and 3 months.

4. RESULTS

The performance measures considered to compare the four rules for safety stock sizing are the actual service level (ASL) and the inventory turnover rate (ITR).

For each group of 30 items with the same demand distribution (i.e. same pattern and parameters, as shown in Table 1) simulation experiments for 10 different values of decision variables of the four sizing rules were computed. Simulation experiments were also taken grouping all items with different parameters but same demand and finally at an even more aggregate level items with the three main demand patterns (i.e. stationary, linear trend and seasonal) were considered together.

In the following figures 1-4 the behaviour of the four safety stock rules for groups of items with the same demand pattern in a stabilised condition and with one month lead times are shown. The third rule, where safety factor is based on service level (see relation 3), gains the best results at this level of aggregation. The four rule performances are similar to the third ones, except when demand has a seasonal pattern: in this case, in fact, both the dynamic sizing rules perform worse than the static ones (see Figure 3).

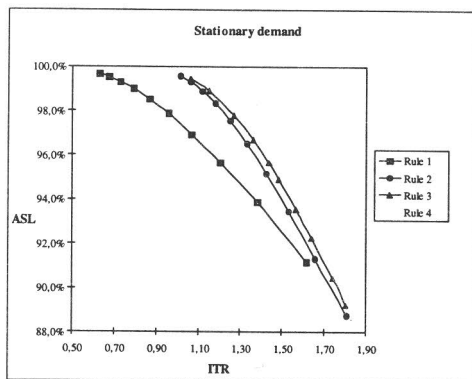


Fig.1 Simulation results for items with stationary demand pattern in a stabilised condition

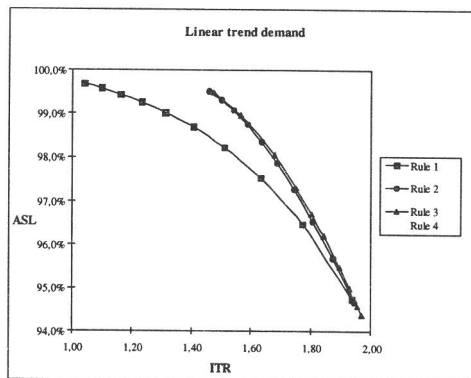


Fig. 2 Simulation results for items with linear trend demand pattern in a stabilised condition

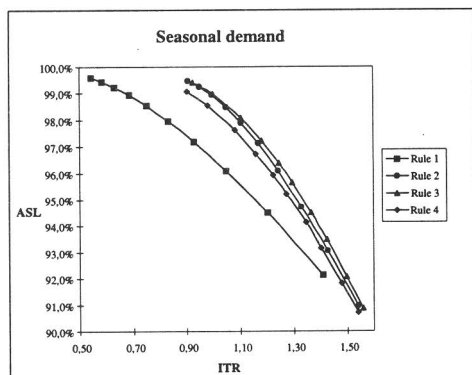


Fig. 3 Simulation results for items with seasonal demand pattern in a stabilised condition

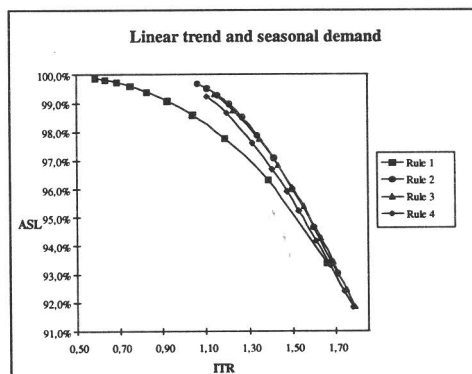


Fig. 4 Simulation results for items with both a linear trend and seasonal demand pattern in a stabilised condition

The first rule, based on periods of supply to be covered (see relation 2), performs worse than the others for all demand patterns (see Figure 1, 2, 3, and 4).

At a more detailed level (i.e. when items with the same parameters of demand distribution are considered) experiments show how the periods of supply rule gets worse and worse with noise component increasing and how the second rule performs better than the third when a seasonal pattern is analysed.

When items with a different demand pattern are instead involved in the same simulation runs the service level static rule gets the best performances, attesting Brown's assertions, followed by the new dynamic rule, the shortage occurrence rule and finally the period of supply rule, as can be seen in Figure 5, where results at the most aggregate level are proposed.

From simulation experiments no relations between the replenishment lead time and the relative performances of safety stock sizing rules were recognised, as can be seen comparing figures 5-7.

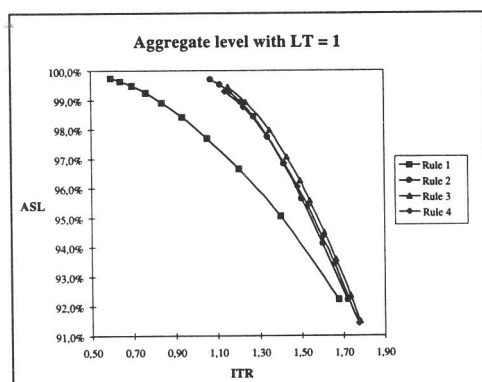


Fig. 5 Simulation results at aggregate level when a Lead Time of one month is considered

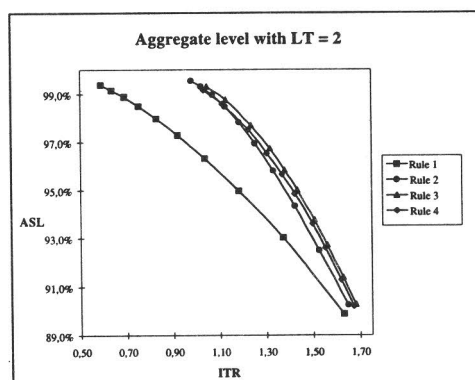


Fig. 6 Simulation results at aggregate level when a Lead Time of two months is considered

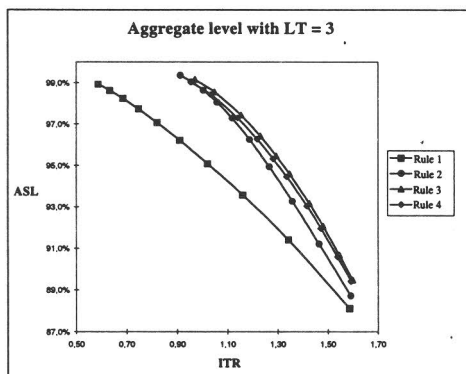


Fig. 7 Simulation results at aggregate level when a Lead Time of three months is considered

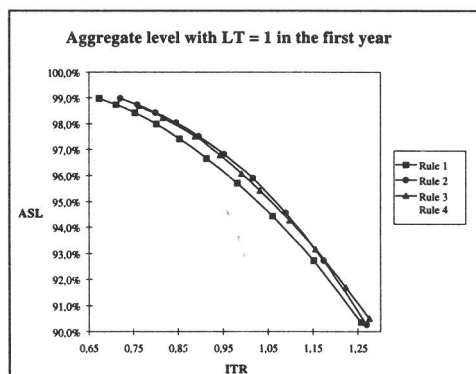


Fig. 8 Simulation results at aggregate level with a one month lead time in the first year of product life

Moving towards more unstable conditions, the difference on performances gained by the four safety stock sizing rules becomes less remarkable (see fig. 8). When a three and one year period are considered, results are similar to those obtained in a stabilised configuration, except that the shortage chance method is better than the service level one for products with a trend demand pattern.

5. CONCLUSIONS

The dynamic sizing method proposed in section 2 doesn't improve system performances as it might be expected; in fact the service level static rule, from which the fourth rule was derived, often gets better results. Thus it looks not worth while including a demand level factor when defining safety stocks.

The worse behaviour of dynamic rules becomes clear especially when analysing a seasonal demand pattern, where major improvements were expected; in this case in fact both the static rule overcome the dynamic ones. Therefore when establishing mechanisms to protect a firm against market uncertainty, it is not correct to make safety stocks variable during the planning horizon, trying to adapt them to demand pattern. If safety stocks, in fact, have to preserve a company from the risk related to the forecasting process, then they depend on the forecast error and thus on the ability of the chosen forecast methodology to follow actual demand patterns rather than on demand level in the replenishment lead time.

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