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ADVANCED MANUFACTURING SYSTEMS AND TECHNOLOGY

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WORK-IN-PROCESS EVALUATION IN JOB-SHOP AND FLEXIBLE MANUFACTURING SYSTEMS: MODELLING AND EMPIRICAL TESTING

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KEY WORDS: work-in-process, lead time, job-shop, flexible manufacturing system

ABSTRACT The authors investigate how work-in-process can be reduced when passing from a job-shop to a flexible manufacturing system. As explained in the paper, the problem can be brought back to analyzing how lead time can be shortened. Lead time evaluation formulas for a job-shop and a flexible manufacturing system are therefore proposed. In the case of job-shop systems the formula is empirically tested by comparing estimated values with data collected in a manufacturing firm. Thus, by comparing the relations proposed, the factors concurring to reduce work-in-process when passing from a job-shop to a flexible manufacturing system are identified: the different ways of processing in the two systems and the possibility of reducing lot size.

1. INTRODUCTION

The authors are particularly interested in investigating how work-in-process can be affected by a change in the production system, i.e. when passing from a job-shop environment to a flexible manufacturing one. In the following sections, evaluation formulas for work-in-process in the two systems analyzed are proposed and finally compared in order to find the factors concurring to reduce its amount.

2. WORK-IN-PROCESS EVALUATION

According to Little's law [1], the following parameters are significant for estimating the value of work-in-process *(WIP)* in a system with a stationary process:

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a_i = average lead time of the i-th item lot produced in the system [days];
m_i = average value of raw materials per lot for the i-th item [$/lot];
l_i = average value of direct costs per lot for the i-th item [$/lot];
ΔT_i = average time between the arrivals of two consecutive item i lots to the system [days/lot].

In fact, if the value is supposed to be added to item lots linearly with time as shown in fig. 1, then [2]:

$$WIP_i = \frac{a_i}{\Delta T_i}\left(m_i + \frac{l_i}{2}\right) \quad [\$] \tag{1}$$

The value of WIP_i is therefore represented by the area of the trapezium in fig. 1. The evaluation formula (1) shows how WIP_i [$] is the product of the average number (Q_i) of lots in the system provided by Little's law $(Q_i = a_i/\Delta T_i)$ and the average value of each lot. On average 50% of the final value of each lot is already present and 50% remains to be added. The most difficult parameter to determine in equation (1) is the average lead time. Hence the reason why lead time evaluation formulas for job-shop and flexible manufacturing systems are proposed in the following sections.

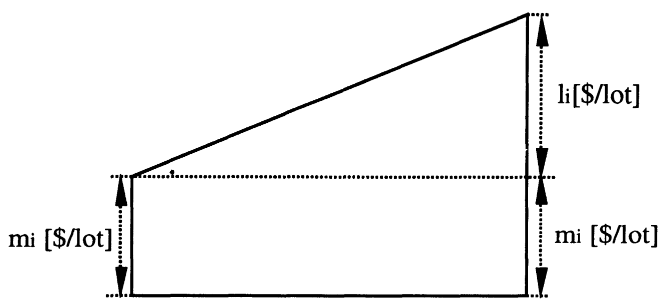

Fig. 1 - WIP when value is added to item lots linearly with time

3. LEAD TIME EVALUATION IN A JOB-SHOP SYSTEM

It would be useful to express lead time as a function of some known parameters relative to process cycles and job-shop characteristics. The authors propose the evaluation formula explained in section 3.1 and provide empirical test results in section 3.2.

3.1 MODELLING

When considering the i-th item belonging to the j-th product family manufactured in the system:

$$a_i \cong \frac{n_i}{X_j} \tag{2}$$

where: a_i = lead time of the i-th item lot [days];

n_i = number of cycle operations for the i-th item [operations/lot];
X_j = average number of operations completed per day on each lot of the j-th family [operations/(lot*day)].

Parameter n_i can be easily deduced from each item routing, while particular attention must be paid to the number of operations completed per day on each lot of the j-th family(X_j).
The first to mention a similar parameter was Corke [3], who talked about the possible utility of the average number of operations per lot and day in lead time evaluation for job-shop systems. He considered this parameter as a characteristic of the whole job-shop, without distinguishing product families, and ascribing it values which ranged between 0.2 and 0.4 operations/(lot*day). Also on the grounds of empirical evidence (as shown in the following section) it appears that when a single value is ascribed to parameter X, no significant errors are made, provided that the items are characterized by a high degree of homogeneity in their manufacturing cycles. When, on the other hand, different families are processed in the job-shop system, X must be related both to the manufacturing system management and the product characteristics.
In fact from relation (2) it follows that:

$$X \cong \frac{n_i}{a_i} = \frac{n_i}{\sum_{h=1}^{n_i} t_h} = \frac{1}{\bar{t}_i} \tag{3}$$

where: t_h = duration of the h-th operation of item i cycle, including run time, set-up time, moving time and queue time [days];
$\bar{t}_i$ = average duration of a cycle operation for the i-th item [days].

Parameter X can therefore be related to the average duration of a cycle operation. However, each machine in the system differs for processing times, work loads and therefore queue length. Thus, lead time depends on which machines are used to process the i-th item. Since items belonging to the same family have similar manufacturing cycles, the error introduced by considering the average duration of an operation for a whole family is less than that given by calculating the average duration of an operation for all the products the system produces. Hence the reason why the authors relate X to item families and not to the whole system as Corke proposes.
From relation (2) it follows that for a given item family, lead times in a job-shop system approximately differ only for the number of cycle operations, i.e. the number of displacements in the system. Lot size is not involved in (2); this is due to the fact that most of the time an item spends in a job-shop system is wasted waiting in queue to be processed, thus independently from its size. Moreover items belonging to the same family are usually manufactured in lots of similar entity. Omitting the variability of time with lot size does not therefore lead to a considerable mistake in the approximation.

3.2 EMPIRICAL TESTING

The authors tested relation (2) in a local firm, which produces centrifugal ventilators and where component parts manufacturing is organized as a job-shop system.
Data on several families of items were collected over a three month period. To obtain the value of X_j for each j-th family every day the number of operations performed for the family and the number of its lots present in the system were measured. Then X was calculated as

the mean of the values obtained each day by dividing the number of operations performed by the number of lots in the system.
The authors furthermore measured the average lead time for each item produced in the observed period and deduced the number of their cycle operations from their routings.
The results of the four most representative families are shown in the tables below, which also provide lead time evaluation when a single value of X (X=0.29) is calculated for the whole system.

Part number	number of operations	Actual Lead Time [days]	Estimated Lead Time [days] X_j=0.34	Error	Estimated Lead Time [days] X=0.29	Error
10100110	3	9.9	8.8	12.5%	10.3	3.9%
11000110	3	9.85	8.8	11.9%	10.3	4.4%
12800110	3	6.6	8.8	25.0%	10.3	35.9%
17130110	3	10.1	8.8	14.8%	10.3	1.9%
20100110	2	5.2	5.9	11.9%	6.9	24.6%
22200110	4	10.3	11.8	12.7%	13.8	25.4%
23200110	4	9.9	11.8	16.1%	13.8	28.3%
25000110	2	6.1	5.9	3.4%	6.9	11.6%
26300110	2	5.75	5.9	2.5%	6.9	16.7%
29000110	2	5.3	5.9	10.2%	6.9	23.2%
average	2.8	7.9	8.24	12.1%	9.64	17.59%

Table 1 - Actual and estimated lead times for side panel family

Part number	number of operations	Actual Lead Time [days]	Estimated Lead Time [days] X_j=0.29	Error	Estimated Lead Time [days] X=0.29	Error
11600130	1	3.2	3.4	5.9%	3.4	5.9%
11800130	1	3.3	3.4	2.9%	3.4	2.9%
12000130	1	3.1	3.4	8.8%	3.4	8.8%
12200130	1	3.8	3.4	11.8%	3.4	11.8%
12800130	1	3.8	3.4	11.8%	3.4	11.8%
13100130	1	3.2	3.4	5.9%	3.4	5.9%
14000130	1	3.4	3.4	0.0%	3.4	0.0%
14500130	1	3.45	3.4	1.5%	3.4	1.5%
15000130	1	3.6	3.4	5.9%	3.4	5.9%
16300130	1	2.5	3.4	26.5%	3.4	26.5%
average	1	3.33	3.4%	8.1%	3.4%	8.1%

Table 2 - Actual and estimated lead time for back panel family

Part number	number of operations	Actual Lead Time [days]	Estimated Lead Time [days] $X_j=0.21$	Error	Estimated Lead Time [days] $X=0.29$	Error
21000200	4	18.8	19.0	1.1%	13.8	36.2%
23190200	3	12.1	14.3	15.4%	10.3	17.5%
24000200	6	21.6	28.6	24.5%	20.7	4.3%
24090200	3	12.2	14.3	14.7%	10.3	18.4%
25090200	3	12.1	14.3	15.4%	10.3	17.5%
26300200	5	23.4	23.8	1.7%	17.2	36.0%
27100200	4	20.9	19.0	10.0%	13.8	51.4%
28000200	4	18.4	19.0	3.2%	13.8	33.3%
29000200	4	18.9	19.0	0.5%	13.8	37.0%
29092200	2	11.8	9.5	24.2%	6.9	71.0%
average	3.8	17.02	18.08	11.07%	13.09	32.26%

Table 3 - Actual and estimated lead times for nozzle family

Part number	number of operations	Actual Lead Time [days]	Estimated Lead Time [days] $X_j=0.82$	Error	Estimated Lead Time [days] $X=0.29$	Error
11000260	6	6.1	7.3	16.4%	20.7	70.5%
12000260	4	5.6	4.9	14.3%	13.8	59.4%
12200260	4	5.6	4.9	14.3%	13.8	59.4%
13100260	6	6.0	4.9	22.4%	20.7	71.0%
18000260	6	6.4	7.3	12.3%	20.7	69.1%
19000260	6	6.0	7.3	17.8%	20.7	71.0%
42091260	3	3.65	3.7	1.4%	10.3	64.6%
43600260	3	3.4	3.7	8.1%	10.3	67.0%
55094260	4	4.0	4.9	18.4%	13.8	71.0%
57194260	4	4.2	4.9	14.3%	13.8	69.6%
average	4.6	5.09	5.38	13.97%	15.86	67.26%

Table 4 - Actual and estimated lead times for fan family

4. LEAD TIME EVALUATION IN A FLEXIBLE MANUFACTURING SYSTEM

For a flexible manufacturing system formed by working centers, the "scheduling factor" *(SF)* can be defined as the average number of hours per day of work capacity that can be dedicated to each production lot [4].
The high investments required for fixtures and tools to provide the system with the maximum degree of flexibility, i.e. the possibility of simultaneously processing items of the same type, lead to limit their availability. Therefore different items must be simultaneously processed in order to saturate the system. Thus, the scheduling factor is calculated by dividing the work capacity of the system with the number of different item lots that have to simultaneously be present.

Like X, also SF must be related to each product family processed by the system, since a different number of fixture and tools can be provided to each family.
Therefore lead time for a flexible manufacturing system can be evaluated in the following way:

$$a_i \cong \frac{r_i}{SF_j} \tag{4}$$

where: a_i = lead time for the i-th item lot [days];
r_i = total average run time for the i-th item, i.e. the total hours of work needed by the system to process a lot of a given size [hours/lot];
SF_j = the scheduling factor for the j-th family [hours/(lot*day)].

A formal comparison between lead time in a job-shop (JS) and in a flexible manufacturing system (FMS) can thus be made.
There appears to be a correspondence between the average number (X_j) of operations per lot performed daily for a given family in a job-shop and the "scheduling factor" (SF_j) for a FMS.
Since in a job-shop environment most of the time is spent waiting for the items to be processed by each machine of the routing, lead time is substantially determined by the number of operations to be performed, i.e. the number of displacements from one centre to another, and can be considered lot size independent (see equation 2). On the other hand, in a flexible manufacturing system inter-operation waiting times are not considerable and lead time appears to be strictly related to lot size (see equation 4). Hence, for a given family lead time is a fixed parameter when a job-shop system is involved, but is considered as variable one when a flexible manufacturing system is analyzed.

5. PASSING FROM A JOB-SHOP TO A FLEXIBLE MANUFACTURING SYSTEM

By analyzing relationship (1), (2) and (4) it is possible to identify which factors concur in reducing work-in-process when passing from a job-shop to a flexible manufacturing system with identical capacity.
Considering item data $(m, l, \Delta T)$ unchanged and omitting, for semplicity, the subscript related to the i-th item being analyzed, it can be written:

$$\frac{WIP_{JS}}{WIP_{FMS}} = \frac{\frac{a_{JS}}{\Delta T}\left(m+\frac{l}{2}\right)}{\frac{a_{FMS}}{\Delta T}\left(m+\frac{l}{2}\right)} = \frac{a_{JS}}{a_{FMS}} = \frac{n/X_j}{r/SF_j} \tag{5}$$

Hence, the reduction of WIP is associated with shorter lead time one has when passing from one system to the other.
A first reduction can be ascribed to the different ways of processing in the two systems: a job-shop has fixed lead times, while a flexible manufacturing system has variable ones, with no queue time between operations (see equation 5). A numeric example is given in table 5, which shows how it is possible to shorten the lead time from the 28 days in the job-shop to the 9 days in the flexible manufacturing system (see columns 2 and 3).
A further reduction can be obtained by reducing lot size, due to the dependent nature of lead time in a flexible manufacturing system. If k is the "size-reducing factor", then lead time can be shortened with the same factor until a single unit lot is reached (see columns 5 and 6 of table 5).

The following relationship, in fact, exists:

$$r_{FMS} = \frac{r_{JS}}{k} \tag{6}$$

	Job-Shop	**FMS** no lot-size reduction (k=1)	**FMS** with lot-size reduction (k=9)	**FMS** with lot-size reduction (k=99)
Independent variables				
S_i = lot size [units]	99	99	99/9=11	99/99=1
n_i [operations/lot]	7	1	1	1
c_i = run unit time [hours]	$0,\overline{63}$	$0,\overline{63}$	$0,\overline{63}$	$0,\overline{63}$
$X_j\left[\frac{\text{operations}}{\text{lot}\cdot\text{day}}\right]$ $SF_j\left[\frac{\text{hours}}{\text{lot}\cdot\text{day}}\right]$	0,25	7	7	7
m_i [$/lot]	54	54	54/9	54/99
l_i [$/lot]	36	36	36/9	36/99
ΔT_i [days/lot]	1	1	1/9	1/99
Dependent variables				
$r_i = S_i \cdot c_i$ [hours/lot]	63	63	63/9	$63/99=0,\overline{63}$
capacity $= \frac{r_i}{\Delta T_i}$ [hours / day]	63	63	63	63
$a_i=\frac{n_i}{X_j}$ [days] $a_i=\frac{r_i}{SF_j}$ [days]	28	9	9/9=1	9/99=0,091
$Q_i=\frac{a_i}{\Delta T_i}$ [lots]	28	9	9	9
$WIP_i=a_i\left[\frac{1}{\Delta T_i}\left(m_i+\frac{l_i}{2}\right)\right]$ [$]	$28 \cdot 72$	$9 \cdot 72$	$9/9 \cdot 72$	$9/99 \cdot 72$

Table 5 - Formal analogy between Job-Shop and FMS

The coefficient of WIP reduction can, therefore, be calculated:

$$\frac{a_{FMS}}{a_{JS}} = \frac{\dfrac{r_{JS}/k}{SF_j}}{\dfrac{n}{X_j}} = \frac{1}{k \cdot SF_j} \cdot \left(\frac{r_{JS} \cdot X_j}{n} \right) \tag{7}$$

The terms in brackets represent characteristics of the job-shop system being abandoned, while k and SF describe the flexible manufacturing system to which one passes and represent the variables granting a WIP reduction. The scheduling factor, in fact, takes into account the different nature of a flexible manufacturing system as compared to a job-shop one, which is instead characterized by the average number of operations per lot and day *(X)*. The size-reducing factor describes the possibility of reducing lot-size thanks to the lower set-up times in FMS.

From equation (7) it can be deduced that minimum lead time, and consequently minimum level of work-in-process, is related to the maximum values of the size-reducing factor *(k)* and the scheduling factor *(SF)*.

The maximum value which can in theory be given to k is equal to lot size [in table 5 $k = 99$]; the actual value assumed by this parameter does not however depend on FMS characteristics but rather on the requirements of the upstream and downstream stages.

On the other hand the value of the scheduling factor is related to the amount of investments on fixtures and tools. If the flexible manufacturing system is provided with the maximum degree of flexibility, than SF is equal to the work capacity of the system.

When trying to limit the investments needed for the FMS, the value of SF that is associated with the same lead time of the job-shop being abandoned must be considered as a lower bound.

It is, therefore, possible to determine the minimum acceptable value for SF as follows:

$$a_{FMS} = a_{JS} \Rightarrow \frac{n}{X_j} = \frac{r/k}{SF_j} \Rightarrow \left(SF_j\right)_{\min} = \frac{r}{k} \cdot \frac{X_j}{n} \tag{8}$$

An investment on fixtures and tools leading to a scheduling factor less than SF_{min} provides the flexible manufacturing system with a lead time which is worse than that of the job-shop.

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